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AUTUMN BREAK HOME WORK
CLASS XI SESSION 2023-2024

Day 1(20/10/23)

CONCEPTS AND RESULTS

** **Set :** a set is a well-defined collection of objects.

If a is an element of a set A, we say that " a belongs to A" the Greek symbol $\in$ (epsilon) is used to denote the phrase 'belongs to'. Thus, we write $a \in A$. If 'b' is not an element of a set A, we write $b \notin A$ and read "b does not belong to A".

There are two methods of representing a set :

(i) Roster or tabular form (ii) Set-builder form.

In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within brackets { }. For example, the set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set. For example, in the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V, we write V = {x : x is a vowel in English alphabet}

** **Empty Set :** A set which does not contain any element is called the empty set or the null set or the void set. The empty set is denoted by the symbol φ or { }.

** **Finite and Infinite Sets :** A set which is empty or consists of a definite number of elements is
called finite otherwise, the set is called infinite.

** **Equal Sets :** Two sets A and B are said to be equal if they have exactly the same elements and we write
$A = B$. Otherwise, the sets are said to be unequal and we write $A \neq B$.

** **Subsets :** A set A is said to be a subset of a set B if every element of A is also an element of B.

In other words, $A \subset B$ if whenever $a \in A$, then $a \in B$. Thus $A \subset B$ if $a \in A \Rightarrow a \in B$

If A is not a subset of B, we write $A \not\subset B$.

** Every set A is a subset of itself, i.e., $A \subset A$.

** φ is a subset of every set.

** If $A \subset B$ and $A \neq B$, then A is called a proper subset of B and B is called superset of A.

** If a set A has only one element, we call it a singleton set. Thus, { a } is a singleton set.

** Closed Interval : $[a , b] = \{x : a \leq x \leq b\}$

** Open Interval : $(a , b) = \{ x : a < x < b\}$

** Closed open Interval : $[a , b) = \{x : a \leq x < b\}$

** Open closed Interval : $(a , b] = \{ x : a < x \leq b\}$

** **Power Set :** The collection of all subsets of a set A is called the power set of A. It is denoted by P(A)

If A is a set with $n(A) = m$, then it can be shown that $n[P(A)] = 2^m$.

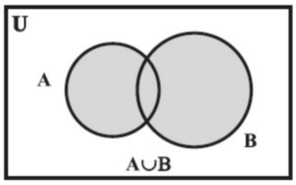
** **Universal Set :** The largest set under consideration is called Universal set.

** **Union of sets :** The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both). In symbols, we write.

$A \cup B = \{ x : x \in A \text{ or } x \in B \}$.

$x \in A \cup B \Rightarrow x \in A \text{ or } x \in B$

$x \notin A \cup B \Rightarrow x \notin A \text{ and } x \notin B$



** **Some Properties of the Operation of Union**

(i) $A \cup B = B \cup A$ (Commutative law)

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

(iii) $A \cup \varphi = A$ (Law of identity element, φ is the identity of $\cup$)

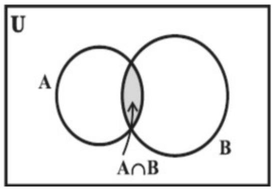
(iv) $A \cup A = A$ (Idempotent law)

(v) $U \cup A = U$ (Law of U)

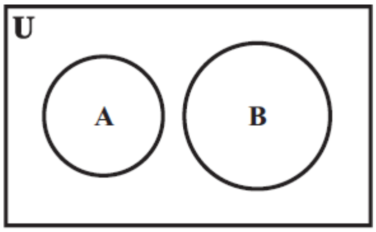
** **Intersection of sets :** The intersection of two sets A and B is the set of all those elements which belong to both A and B. Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$x \in A \cap B \Rightarrow x \in A \text{ and } x \in B$

$x \notin A \cap B \Rightarrow x \notin A \text{ or } x \notin B$



** **Disjoint sets :** If A and B are two sets such that $A \cap B = \varphi$, then A and B are called disjoint sets.



** **Some Properties of Operation of Intersection**

(i) $A \cap B = B \cap A$ (Commutative law).

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).

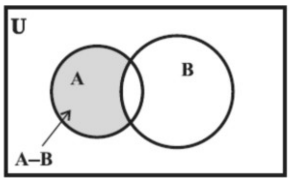
(iii) $\varphi \cap A = \varphi$, $U \cap A = A$ (Law of φ and U).

(iv) $A \cap A = A$ (Idempotent law)

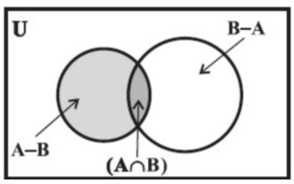
(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) i. e., $\cap$ distributes over $\cup$

** **Difference of sets :** The difference of the sets A and B in this order is the set of elements which belong to A but not to B. Symbolically, we write A – B and read as " A minus B".

$A - B = \{ x : x \in A \text{ and } x \notin B \}$.

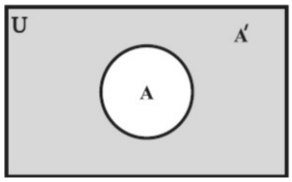


* The sets $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets, i.e., the intersection of any of these two sets is the null set.



** **Complement of a Set :** Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A' to denote the complement of A with respect to U.

Thus, $A' = \{x : x \in U \text{ and } x \notin A\}$. Obviously $A' = U - A$



** **Some Properties of Complement Sets**

1. Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \varphi$
2. De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
3. Law of double complementation : $(A')' = A$
4. Laws of empty set and universal set $\varphi' = U$ and $U' = \varphi$.

** **Practical Problems on Union and Intersection of Two Sets :**

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B)$, if $A \cap B = \varphi$.

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

QUESTIONS FOR HHW

1 : How many elements are there in the complement of set A?

A. 0 B. 1 C. All the elements of A D. None of these

2 : Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, P = {1, 2, 5}, Q = {6, 7}. Then P ∩ Q' is :

A. P B. Q C. Q' D. None

3. The cardinality of the power set of {x: x∈ N, x≤10} is______.

A. 1024 B. 1023 C. 2048 D. 2043

4. If A, B and C are any three sets, then A × (B ∪ C) is equal to:

A. $(A \times B) \cup (A \times C)$
B. $(A \cup B) \times (A \cup C)$
C. $(A \times B) \cap (A \times C)$
D. None of the above

<u>(B) ASSERTION AND REASON</u>

1. **DIRECTION**: In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.
(d) Assertion is false but reason is true
1. Assertion (A) 'The collection of all natural numbers less than 100' is a set.
Reason (R) :A set is a well-defined collection of the distinct objects.

Day 2 (21/10/23)

RELATIONS & FUNCTIONS
CONCEPTS AND RESULTS

** **Cartesian Products of Sets :** Given two non-empty sets P and Q. The cartesian product P × Q is the set of all ordered pairs of elements from P and Q, i.e., $P \times Q = \{ (p , q) : p \in P, q \in Q \}$

** Two ordered pairs are equal, if and only if the corresponding first elements, are equal and the second elements are also equal.

** If there are p elements in A and q elements in B, then there will be pq elements in A × B, i.e. if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

** If A and B are non-empty sets and either A or B is an infinite set, then so is A × B.

** $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.

** **Relation :** A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B. The second element is called the image of the first element.

** The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

** The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the co-domain of the relation R. Range ⊆ co-domain.

** A relation may be represented algebraically either by the Roster method or by the Set-builder method.

** An arrow diagram is a visual representation of a relation.

** The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and the total number of relations is 2^{pq}.

** A relation R from A to A is also stated as a relation on A.

** **Function:** A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

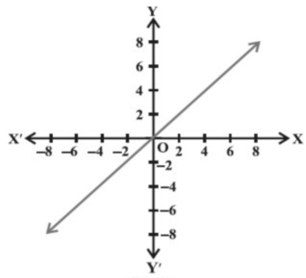
In other words, a function f is a relation from a non-empty set A to a non-empty set B such that
the domain of f is A and no two distinct ordered pairs in f have the same first element.

If f is a function from A to B and $(a, b) \in f$, then f (a) = b, where b is called the image of a under
f and a is called the pre-image of b under f.

** A function which has either R or one of its subsets as its range is called a real valued function. Further,
if its domain is also either R or a subset of R, it is called a real function.

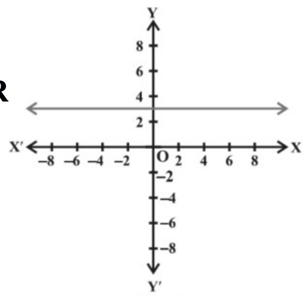
Some functions and their graphs

** **Identity function** Let **R** be the set of real numbers. Define the real valued function f : **R** →**R** by y = f(x) = x for each $x \in$ **R**. Such a function is called the identity function. Here the domain and range of f are **R**.



f(x) = x

Constant function : Define the function f: **R** → **R** by y = f (x) = c, $x \in$ **R** where c is a constant and each $x \in$ **R**. Here domain of f is **R** and its range is {c}.

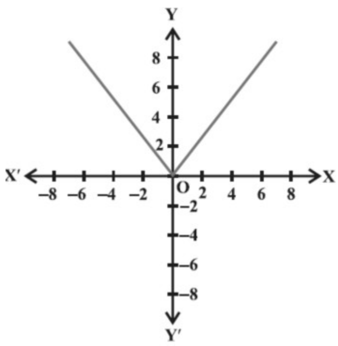


f(x) = 3

Polynomial function : A function f : **R**→**R** is said to be polynomial function if for each x in **R**,
$y = f(x) = a_0 + a_1x + a_2x^2 + ... + a_n x^n$, where n is a non-negative integer and $a_0, a_1, a_2, ..., a_n \in$ **R**.

** **Rational functions :** are functions of the type $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomial functions of x defined in a domain, where $g(x) \neq 0$.

** **The Modulus function :** The function f: **R**→**R** defined by f(x) = |x| for each $x \in$ **R** is called modulus function. For each non-negative value of x, f(x) is equal to x.
But for negative values of x, the value of f(x) is the negative of the value of x, i.e., $f(x) = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$.

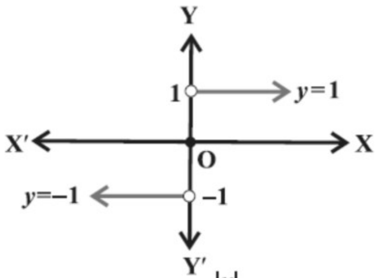


f(x) = |x|

** **Signum function :** The function f : **R**→**R** defined by

$$f(x)=\begin{cases}1, & \text{if } x>0\\ 0, & \text{if } x=0\\ -1, & \text{if } x<0\end{cases}$$

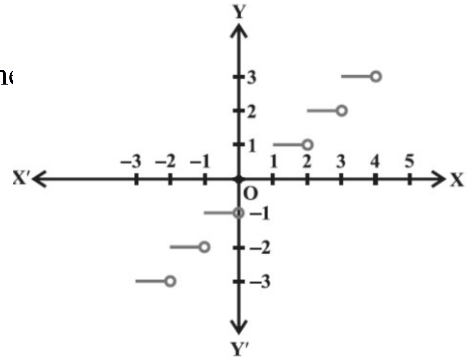
is called the signum function. The domain of the signum function is **R** and the range is the set {–1, 0, 1}.



$f(x)=\frac{|x|}{x}$

** **Greatest integer function :**
The function f : **R** → **R** defined by f(x) = [x], x∈**R** assume the value of the greatest integer, less than or equal to x. Such a function is called the greatest integer function.

[x] = –1 for $-1 \le x < 0$
[x] = 0 for $0 \le x < 1$
[x] = 1 for $1 \le x < 2$
[x] = 2 for $2 \le x < 3$ and so on.



f(x) = [x]

Algebra of real functions
** **Addition of two real functions :** Let f : X → **R** and g : X → **R** be any two real functions, where X ⊂ **R**.
Then, we define (f + g): X → **R** by (f + g) (x) = f (x) + g (x), for all x∈X.
** **Subtraction of a real function from another :** Let f :X → **R** and g:X → **R** be any two real functions,
where X⊂**R**. Then, we define (f – g) : X→**R** by (f – g) (x) = f(x) – g(x), for all x∈X.
** **Multiplication by a scalar :** Let f : X→**R** be a real valued function and α be a scalar. Here by scalar, we
mean a real number. Then the product α f is a function from X to **R** defined by
(α f) (x) = α f (x), x∈X.
** **Multiplication of two real functions :** The product (or multiplication) of two real functions f :X→**R** and
g :X→**R** is a function fg :X→**R** defined by (fg) (x) = f(x) g(x), for all x∈X.
** **Quotient of two real functions** Let f and g be two real functions defined from X→**R** where X ⊂ **R**. The
quotient of f by g denoted by $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$, provided g(x) ≠ 0, x ∈ X

QUESTIONS FOR HHW

Q 1. If A= {a, b} and B = {1, 2} then the number of functions from set A to set B is
(A) 2 (B) 4 (C) 16 (D) None of these

Q 2. A function is defined by f(t) = 2t – 5, then the value of f(-3) is

(A) - 11 (B) 11 (C) 1 (D) -1

Q 3. If f(x) = - |x|. Choose the correct option from the following:

(A) Domain is set of negative real numbers (B) Range is set of real numbers

(C) Range is set of all negative integers (D) Range is $(-\infty, 0]$

Q 4. Let f = {(1, 1), (2, 3), (0, -1), (-1, -3)} be a function from Z to Z defined by f(x) = mx + c.

Determine c.

(A) 1 (B) 0 (C) - 1 (D) - 3

Assertion and Reason type (1 mark)

Q 5.In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

ASSERTION (A): The function f: A→B defined by f = {(1, x), (2, y), (3, x)}, then its domain is A = {1, 2, 3} and range is {x, y}.
REASON(R) : The range of the function f is always the co-domain set.

Day 3(22/10/23)

TRIGONOMETRIC FUNCTIONS
CONCEPTS AND RESULTS

Angles : Angle is a measure of rotation of a given ray about its initial point.

**** <u>Measurement of an angle</u>.**

****English System (Sexagesimal system)**

(i) 1 right angle = 90 degrees = 90^o. (ii) 1^o = 60 minutes = 60’. (iii) 1’ = 60 second = 60’’.

****French System (Centesimal system)**

(iv) 1 right angle = 100 grades = 100 g. (v) 1 g = 100 minutes = 100 ‘ (vi) 1’ = 100 seconds = 100 ‘’

****Circular System.**

(vii) $180^o = 200^g = \pi$ radians = 2 right angles, where a radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

(viii) The circular measure θ of an angle subtended at the centre of a circle by an arc of length **l** is equal to the ratio of the length **l** to the radius r of the circle.

(ix) Each interior angle of a regular polygon of n sides is equal to $\frac{2n-4}{n}$ right angles.

T-ratios	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π

Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	n.d	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

** **Formulae for t-ratios of Allied Angles :**

All T-ratio changes in $\frac{\pi}{2} \pm \theta$ and $\frac{3\pi}{2} \pm \theta$ while remains unchanged in $\pi \pm \theta$ and $2\pi \pm \theta$.

$\sin\left(\frac{\pi}{2} \pm \theta\right) = \cos\theta$ $\qquad$ $\sin\left(\frac{3\pi}{2} \pm \theta\right) = -\cos\theta$

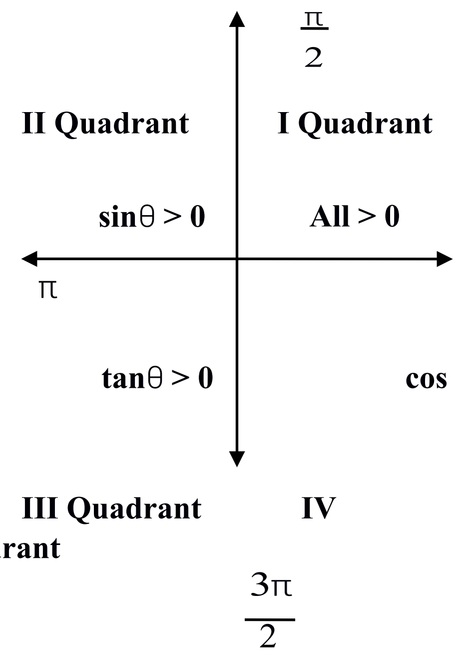
$\cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin\theta$ $\qquad$ $\cos\left(\frac{3\pi}{2} \pm \theta\right) = \pm\sin\theta$

$\tan\left(\frac{\pi}{2} \pm \theta\right) = \mp\cot\theta$ $\qquad$ $\tan\left(\frac{3\pi}{2} \pm \theta\right) = \mp\cot\theta$

$\sin(\pi \pm \theta) = \mp\sin\theta$ $\qquad$ $\sin(2\pi \pm \theta) = \pm\sin\theta$

$\cos(\pi \pm \theta) = -\cos\theta$ $\qquad$ $\cos(2\pi \pm \theta) = \cos\theta$

$\tan(\pi \pm \theta) = \pm\tan\theta$ $\qquad$ $\tan(2\pi \pm \theta) = \pm\tan\theta$



** **Sum and Difference formulae :**

sin(A + B) = sin A cos B + cos A sin B

sin(A – B) = sin A cos B – cos A sin B

cos(A + B) = cos A cos B – sin A sin B

cos(A – B) = cos A cos B + sin A sin B

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$, $\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$

$\tan\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}$, $\cot(A+B) = \frac{\cot A . \cot B - 1}{\cot B + \cot A}$, $\cot(A-B) = \frac{\cot A . \cot B + 1}{\cot B - \cot A}$

$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

$\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

****Formulae for the transformation of a product of two circular functions into algebraic sum of two circular functions and vice-versa.**

2 sinA cos B = sin (A + B) + sin(A – B)

2 cosA sin B = sin (A + B) – sin(A – B)

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$
$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$, $\quad \sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$.

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$, $\quad \cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$.

** **Formulae for t-ratios of multiple and sub-multiple angles** :

$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$.

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \frac{1-\tan^2 A}{1+\tan^2 A}$

$1 + \cos 2A = 2\cos^2 A$ $\quad 1 - \cos 2A = 2\sin^2 A$ $\quad 1 + \cos A = 2\cos^2\frac{A}{2}$ $\quad 1 - \cos A = 2\sin^2\frac{A}{2}$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$, $\quad \tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$.

$\sin 3A = 3\sin A - 4\sin^3 A$, $\quad \cos 3A = 4\cos^3 A - 3\cos A$

$\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$. & $\cos 15^\circ = \sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$,

$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3} = \cot 75^\circ$ & $\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2 + \sqrt{3} = \cot 15^\circ$.

$\sin 18^\circ = \frac{\sqrt{5}-1}{4} = \cos 72^\circ$ and $\cos 36^\circ = \frac{\sqrt{5}+1}{4} = \sin 54^\circ$.

$\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^\circ$ and $\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72^\circ$.

$\tan\left(22\frac{1}{2}\right)^\circ = \sqrt{2} - 1 = \cot\left(67\frac{1}{2}\right)^\circ$ and $\tan\left(67\frac{1}{2}\right)^\circ = \sqrt{2} + 1 = \cot\left(22\frac{1}{2}\right)^\circ$.

** **Properties of Triangles** : In any Δ ABC,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ [Sine Formula]

$\cos A = \frac{b^2+c^2-a^2}{2bc}$, $\cos B = \frac{c^2+a^2-b^2}{2ca}$, $\cos C = \frac{a^2+b^2-c^2}{2ab}$.

** **Projection Formulae** : $a = b\cos C + c\cos B$, $\quad b = c\cos A + a\cos C$, $\quad c = a\cos B + b\cos A$

** Some important trigonometric substitutions :

$\sqrt{a^2 + x^2}$	Put x = a tan θ or a cot θ
$\sqrt{x^2 - a^2}$	Put x = a sec θ or a cosec θ
$\sqrt{a + x}$ or $\sqrt{a - x}$ or both	Put x = a cos2 θ
$\sqrt{a^n + x^n}$ or $\sqrt{a^n - x^n}$ or both	Put $x^n = a^n \cos 2\theta$
$\sqrt{1 + \sin 2\theta}$	$= \sin\theta + \cos\theta$
$\sqrt{1 - \sin 2\theta}$	$= \cos\theta - \sin\theta,\ 0 < \theta \le \frac{\pi}{4}$
	$= \sin\theta - \cos\theta,\ \frac{\pi}{4} < \theta < \frac{\pi}{2}$

General solutions:

$*\cos\theta = 0 \Rightarrow \theta = n\pi,\quad n \in Z$

$*\sin\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2},\ n \in Z$

$*\tan\theta = 0 \Rightarrow \theta = n\pi,\ n \in Z$

$*\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha,\ n \in Z$

$*\ \cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha,\ n \in Z$

$*\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha,\ n \in Z$

Examples:

1. The value of tan 1° tan 2° tan 3° … tan 89° is:

(a) 0 (b) 1 (c) ½ (d) Not defined

Correct option: (b) 1

Solution: tan 1° tan 2° tan 3° … tan 89°

= [tan 1° tan 2° … tan 44°] tan 45°[tan (90° – 44°) tan (90° – 43°)… tan (90° – 1°)]

= [tan 1° tan 2° … tan 44°] [cot 44° cot 43°……. cot 1°] × [tan 45°]

= [(tan 1° × cot 1°) (tan 2° × cot 2°) …..(tan 44° × cot 44°)] × [tan 45°]

We know that, tan A × cot A =1 and tan 45° = 1

Hence, the equation becomes as;

= 1 × 1 × 1 × 1 × …× 1= 1 {As $1^n = 1$}

2. If α + β = π/4, then the value of (1 + tan α) (1 + tan β) is :

(a) 1 (b) 2 (c) – 2 (d) Not defined

Correct option: (b) 2

Solution: Given, α + β = π/4

Taking "tan" on both sides,

tan(α + β) = tan π/4

We know that, tan(A + B) = (tan A + tan B)/(1 – tan A tan B)

and tan π/4 = 1.

So, (tan α + tan β)/(1 – tan α tan β) = 1

tan α + tan β = 1 – tan α tan β

tan α + tan β + tan α tan β = 1….(i)

(1 + tan α)(1 + tan β) = 1 + tan α + tan β + tan α tan β

= 1 + 1 [From (i)]=2

3. Find the radius of the circle in which a central angle of 60° intercepts an arc of length

37.4 cm (use π = 22/7).
Solution: Given, Length of the arc = l = 37.4 cm
Central angle = θ = 60° = 60π/180 radian = π/3 radians
We know that,r = l/θ
= (37.4) * (π / 3)= (37.4) / [22 / 7 * 3]= 35.7 cm

Q4.**Find the value of √3 cosec 20° – sec 20°.**
Solution:√3 cosec 20° – sec 20°

$$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} \quad = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°} = 4\left(\frac{\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°}{2\sin 20° \cos 20°}\right)$$

$$= 4\left(\frac{\sin 60° \cos 20° - \cos 60° \sin 20°}{\sin 40°}\right) \quad = 4\left(\frac{\sin(60° - 20°)}{\sin 40°}\right) = 4$$

QUESTIONS FOR HHW

1. A horse is tied to a post by a ripe. If the horse moves along a circular path always keeping the rope tight and describes 88 m when it has traced out 72^0 at the centre, find the length of the rope.
2. If Cos θ = $-\frac{1}{2}$, $\pi < \theta < \frac{3\pi}{2}$, Evaluate 4 $\tan^2\theta - 3\,\mathrm{Cosec}^2\,\theta$.
3. Show that $\cos 60^0 + \cos 120^0 + \cos 240^0 - \sin 330^0 = 0$
4. Show that $\sqrt{2+\sqrt{2+2\cos 4x}} = 2\,Cos x$
5. Show that $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$
6. Show that $\cos 2\theta \cos\frac{\theta}{2} - \cos 3\theta \cos\frac{9\theta}{2} = \sin 5\theta \sin\frac{5\theta}{2}$
7. Show that $\frac{1 + Sin\,2x - Cos\,2x}{1 + Sin\,2x + Cos\,2x} = \tan x$
8. Show that Cos A. Cos (60 – A). Cos (60 + A) = $\frac{\cos 3A}{4}$
9. If Cos A + Cos B = [illegible],Sin A + Sin B = [illegible],Show that tan [illegible] = [illegible]
10. Show that 2 $\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = \cos 2\alpha$.

DAY 4(23/10/23)

COMPLEX NUMBERS
CONCEPTS AND RESULTS

* A number of the form (a + ib) where a b $\in$ R, the set of real numbers, and i = $\sqrt{-1}$ (iota) is called a
complex number. It is denoted by z, z = a + ib. "a" is called the real part of complex number z and "b"
is the imaginary part i.e. Re(z) = a and Im(z) = b.

* Two complex numbers are said to be equal i.e. $z_1 = z_2$.

$\Leftrightarrow$ (a + ib) = (c + id)

$\Leftrightarrow$ a = c and b = d

$\Leftrightarrow$ $Re(z_1) = Re(z_2)$ & $Im(z_1) = Im(z_2)$.

* A complex number z is said to be purely real if Im(z) = 0
and is said to be purely imaginary if Re(z) = 0.

* The set R of real numbers is a proper subset of the set of complex number C, because every real number
can be considered as a complex number with imaginary part zero.

* $i^{4n} = (i^4)^n = (1)^n = 1$ $\quad$ $i^{4n+1} = i^{4n}.i = (1).i = i$

$i^{4n+2} = i^{4n}.\ i^2 = (1)(-1) = -1$ $\quad$ $i^{4n+3} = i^{4n}.i^3 = (1)(-i) = -i$.

Algebra of Complex Numbers

** **Addition of two complex numbers :** Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers.
Then, the sum $z_1 + z_2$ is defined as follows: $z_1 + z_2 = (a + c) + i(b + d)$, which is again a complex number.

The addition of complex numbers satisfy the following properties:

(i) The closure law The sum of two complex numbers is a complex number, i.e., $z_1 + z_2$ is a complex number for all complex numbers z_1 and z_2.

(ii) The commutative law For any two complex numbers z_1 and z_2, $z_1 + z_2 = z_2 + z_1$

(iii) The associative law For any three complex numbers z_1, z_2, z_3, $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$.

(iv) The existence of additive identity There exists the complex number 0 + i 0 (denoted as 0), called the additive identity or the zero complex number, such that, for every complex number z, z + 0 = z.

(v) The existence of additive inverse To every complex number z = a + ib, we have the complex number – a + i(– b) (denoted as – z), called the additive inverse or negative of z. Thus z + (–z) = 0 (the additive identity).

** **Difference of two complex numbers :** Given any two complex numbers z_1 and z_2, the difference $z_1 - z_2$ is defined as follows: $z_1 - z_2 = z_1 + (-z_2)$.

** **Multiplication of two complex numbers :** Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers.
Then, the product $z_1 z_2$ is defined as follows: $z_1 z_2 = (ac - bd) + i(ad + bc)$

**The multiplication of complex numbers possesses the following properties :

(i) **The closure law** The product of two complex numbers is a complex number, the product $z_1 z_2$ is a complex number for all complex numbers z_1 and z_2.

(ii) **The commutative law** For any two complex numbers z_1 and z_2, $z_1 z_2 = z_2 z_1$

(iii) **The associative law** For any three complex numbers z_1, z_2, z_3, $(z_1 z_2) z_3 = z_1 (z_2 z_3)$.

(iv) **The existence of multiplicative identity** There exists the complex number $1 + i\,0$ (denoted as 1), called the multiplicative identity such that $z.1 = z$, for every complex number z.

(v) **The existence of multiplicative inverse** For every non-zero complex number $z = a + ib$ or $a + bi$ $(a \neq 0, b \neq 0)$, we have the complex number $\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}$ (denoted by $\frac{1}{z}$ or z^{-1}), called the multiplicative inverse of z such that $z.\frac{1}{z} = 1$ (the multiplicative identity).

(vi) **The distributive law** For any three complex numbers z_1, z_2, z_3,

(a) $z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$ (b) $(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$

****Division of two complex numbers :** Given any two complex numbers z_1 and z_2, where $z_2 \neq 0$, the quotient $\frac{z_1}{z_2}$ is defined by $\frac{z_1}{z_2} = z_1 . \frac{1}{z_2}$

****Modulus a Complex Number :** Let $z = a + ib$ be a complex number. Then, the modulus of z, denoted by $|z|$, is defined to be the non-negative real number $\sqrt{a^2+b^2}$, i.e., $|z| = \sqrt{a^2+b^2}$

**** Properties of Modulus** :

If z, z_1, z_2 are three complex numbers then

(i) $|z| = 0 \Leftrightarrow z = 0$ i.e., real part and imaginary part are zeroes.

(ii) $|z| = |\bar{z}| = |-z|$

(iii) $z.\bar{z} = |z|^2$

(iv) $|z_1.z_2| = |z_1|.|z_2|$

(v) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, \quad z_2 \neq 0$

(vi) $|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2\mathrm{Re}(z_1\bar{z}_2)$

(vii) $|z_1-z_2|^2 = |z_1|^2 + |z_2|^2 - 2\mathrm{Re}(z_1\bar{z}_2)$

(viii) $|z_1+z_2|^2 + |z_1-z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

****Conjugate of a Complex Number :** Let $z = a + ib$ then its conjugate is denoted by $\bar{z} = (a - ib)$.

****Properties of conjugates** :

(i) $\overline{(\bar{z})} = z$

(ii) $z + \bar{z} = 2\mathrm{Re}(z)$

(iii) $z - \bar{z} = 2i\mathrm{Im}(z)$

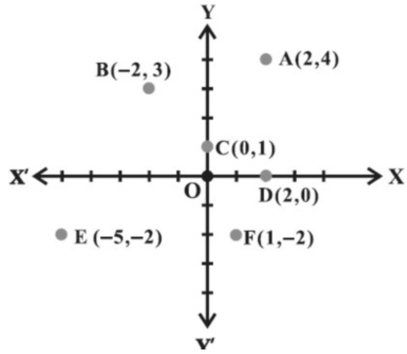
(iv) $z + \bar{z} = 0 \Rightarrow$ z is purely real.

(v) $z.\bar{z} = [Re(z)]^2 + [Im(z)]^2$.

(vi) $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

(vii) $\overline{z_1.z_2} = \overline{z_1}.\overline{z_2}$

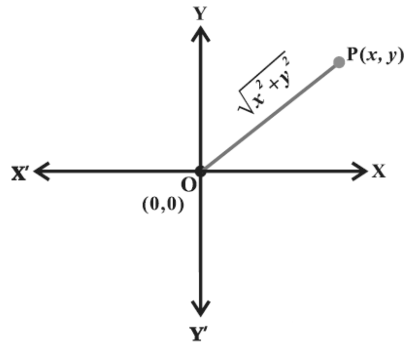
(viii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$, $z_2 \neq 0$



****Argand Plane and Polar Representation**

Some complex numbers such as $2 + 4i$, $-2 + 3i$, $0 + 1i$, $2 + 0i$, $-5 - 2i$ and $1 - 2i$ which correspond to the ordered pairs (2, 4), (−2, 3), (0, 1), (2, 0), (−5, −2), and (1, −2), respectively, have been represented geometrically by the points A, B, C, D, E, and F, respectively.

The plane having a complex number assigned to each of its point is called the complex plane or the Argand plane.

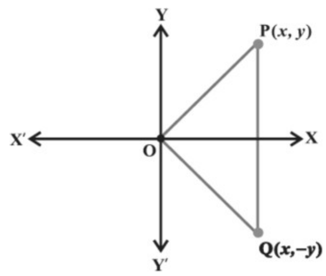


In the Argand plane, the modulus of the complex number $x + iy = \sqrt{x^2 + y^2}$ is the distance between the point P(x, y) to the origin O (0, 0). The points on the x-axis corresponds to the complex numbers of the form $a + i\,0$ and the points on the y-axis corresponds to the complex numbers of the form $0 + i\,b$.

The x-axis and y-axis in the Argand plane are called, respectively, the real axis and the imaginary axis.

The representation of a complex number $z = x + iy$ and its conjugate $z = x - iy$ in the Argand plane are, respectively, the points P (x, y) and Q (x, − y).

Geometrically, the point (x, − y) is the mirror image of the point (x, y) on the real axis



QUESTIONS FOR HHW

1	$i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is equal to (a)0 (b) 1 (c) -1 (d) 2
2	If $z_1 = 3+2i$ and $z_1 = 2-4i$ and $\lvert z_1+z_2\rvert^2 + \lvert z_1-z_2\rvert^2$ is equal (a) 11 (b)22 (c) 55 (d)66
3	The real part of $\frac{(1+i)^2}{3-i}$ is (a)$\frac{1}{5}$ (b)$-\frac{1}{5}$ (c)$\frac{-1}{5}$ (d) None of these
4	If $z = -5i^{-15} - 6i^{-8}$ then $\bar{z}$ is equal to (a) -6-5i (b) -6 +5i (c) 6 -5i (d) 6 +5i
5	Multiplicative Inverse of complex number (1-2i)=... (a)$\frac{1}{5}+i\frac{2}{5}$ (b)$\frac{1}{5}-i\frac{2}{5}$ (c)$-\frac{1}{5}+i\frac{2}{5}$ (d) None of these
6	**Assertion-Reason** Assertion: The equation $ix^2 - 3ix + 2i = 0$ has non real roots.

	Reason: If a, b, c are real and $b^2-4ac\geq0$, then the roots of the equation $ax^2+bx+c=0$ are real and if $b^2-4ac<0$, then the roots of the equation $ax^2+bx+c=0$ are non-real. (a) A is true, R is true; R is a correct explanation of A. (b) A is true, R is true; R is not a correct explanation of A. (c) A is true; R is false (d) A is false; R is true.
7	**Case study** The conjugate of a complex number z is the complex number obtained by replacing i with -i number. It is denoted by $\bar{z}$ The modulus of a complex number $z=a+ib$ is defined as the non-negative real number $\sqrt{a^2+b^2}$. It is denoted by $\lvert z\rvert$ i.e $\lvert z\rvert=\sqrt{a^2+b^2}$ (a) If $(x-iy)(3+5i)$ is the conjugate of $-6-24i$, the find the value of x + y (b) If $f(z)=\frac{7-z}{1-z^2}$, where z = 1 + 2i then find $\lvert f(z)\rvert$.
8	Express the following complex number in the form $a+ib$ $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$
9	Evaluate $1+i^2+i^4+i^6+\cdots+i^{20}$.
10	If z_1, z_2 are $1-i$ and $-2+4i$ respectively find $\operatorname{Im}\left(\frac{z_1z_2}{\bar{z}_1}\right)$
11	Find the value of $(1+i)^6+(1-i)^3$
12	Solve the equation $\lvert z+1\rvert=z+2(1+i)$
13	If $z=x+iy$ and $w=\frac{1-iz}{z-i}$ and $\lvert w\rvert=1$ then show that z is purely real.
14	If x+iy= $\sqrt{\frac{a+ib}{c+id}}$, then prove that $(x^2+y^2)^2=\frac{a^2+b^2}{c^2+d^2}$

DAY 5(24/10/23)

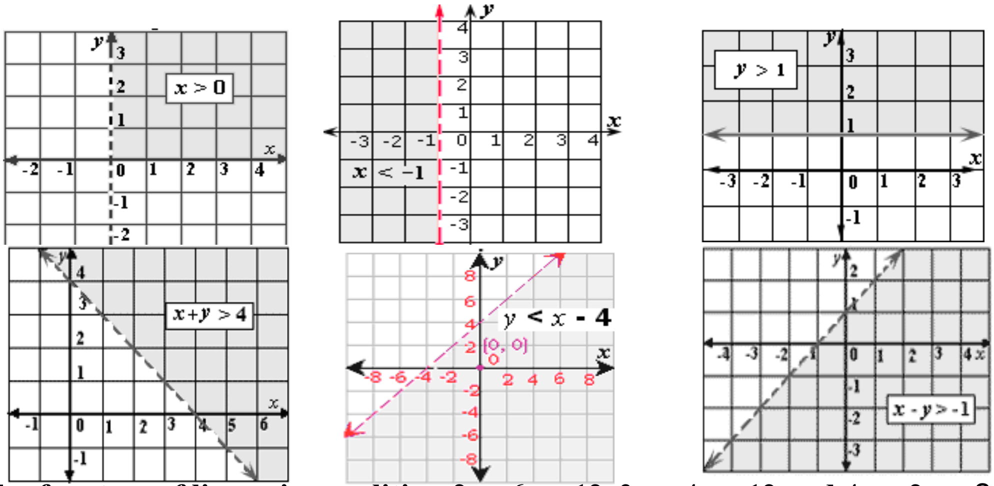
LINEAR INEQUALITIES

MAIN CONCEPTS AND RESULTS

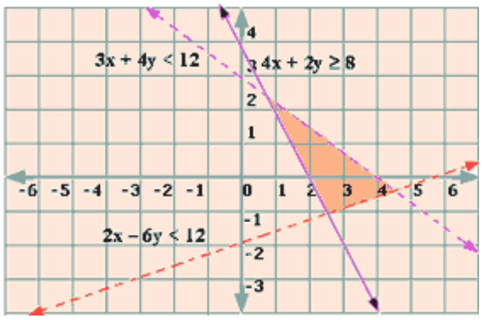
* Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' form an **inequality**.
* **Numerical inequalities :** $3 < 5$; $7 > 5$
* **Literal inequalities :** $x < 5$; $y > 2$; $x \geq 3$, $y \leq 4$
* **Double inequalities :** $3 < 5 < 7$, $2 < y < 4$
* **Strict inequalities :** $ax + b < 0$, $ax + b > 0$, $ax^2 + bx + c > 0$

* **Slack inequalities :** $ax + by \le c$, $ax + by \ge c$, $ax^2 + bx + c \le 0$
* **Linear inequalities :** $ax + b < 0$, $ax + b \ge 0$
* **Quadratic inequalities :** $ax^2 + bx + c > 0$, $ax^2 + bx + c \le 0$

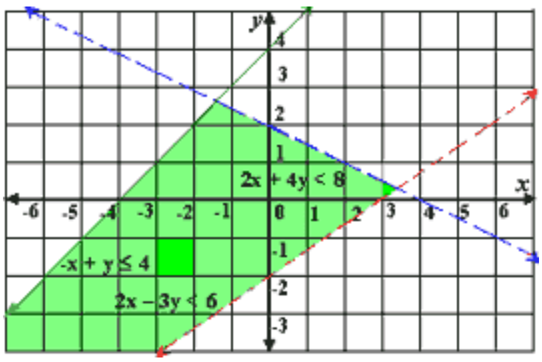
** **Algebraic Solutions of Linear Inequalities in One Variable and their Graphical Representation**



** **Graph of system of linear inequalities,** $2x - 6y < 12$, $3x + 4y < 12$ **and** $4x + 2y \ge 8$.



****Graph the system of linear inequalities.** $2x - 3y < 6$, $-x + y \le 4$, $2x + 4y < 8$



QUESTIONS FOR HHW

1. Given that x,y and b are real numbers and x<y, b >0 , then

A. $\frac{x}{b} < \frac{y}{b}$ B. $\frac{x}{b} \le \frac{y}{b}$ C. $\frac{x}{b} > \frac{y}{b}$ D. $\frac{x}{b} \ge \frac{y}{b}$

2. The solution set of equation $|x+2| \le 5$ is

A. (-7,5) B. [-7.3] C. [-5,5] D. (-7,3)

3. The shaded part of a line is in given figure can also be described as

A. $(-\infty,1)\cup(2,\infty)$ B. $(-\infty,1]\cup[2,\infty)$ C.(1,2) D. [1,2]

4. A recharger manufacturing company produces rechargers and its cost function for a week is $C(x) = \frac{1}{10}(4270 + 23x)$ and its revenue function is $R(x) = 3x$, where x is the number of rechargers produced and sold per week. Number of rechargers must be sold for the company to realize a profit is

A. $x \geq 618$ B. $x > 610$ C. $x > 480$ D.None of These

In the following questions, a statement of Assertion(A)is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d)(A) is false but(R) is true.

5. Assertion (A): The solution set of the inequality $x-3<2$,$x \in N$ is {1,2,3,4,5,6,7,8}.

Reason (R) :Solution set of a inequality in x is set of values of x satisfying the inequality .

Answer . 1.A 2.B. 3. A 4. B 5. d

Short type questions (2marks/3 marks)

1. Solve the inequation $3x + 17 \leq 2(1-x)$

2. Solve the inequality $\frac{x+3}{x-2} \leq 2$

3. Find all pair of consecutive odd integers , both are smaller than 18, such that their sumis more than20.

4. In a game, a person wins if he gets the sum greater than 20 in four throws of a die. In three throws he got numbers 6, 5, 4. What should be number in his fourth throw, so that he wins the game.

5. Solve the inequalities and represent the solution graphically on number line:
$5x + 1 > -24$, $5x - 1 < 24$.

6. Solve $3x-5 < x+1$.Show the solution on number line.

7. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 liters of 9% acid solution, how many liters of 3% solution will have to be added?

8. Solve the inequality $\frac{2x-3}{4}+9 \geq 3+\frac{4x}{3}$.

9. The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm then find the minimum length of the shortest side.

10. Solve the inequation $\frac{2x+4}{x-1} \geq 5$

DAY 6(25/10/23)

PERMUTATIONS AND COMBINATIONS
MAIN CONCEPTS AND RESULTS

** Fundamental principle of counting, or(the multiplication principle): "If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is m × n."

****Factorial notation** The notation n! represents the product of first n natural numbers, i.e., the product

$1 \times 2 \times 3 \times \ldots \times (n-1) \times n$ is denoted as n!. We read this symbol as 'n factorial'.

Thus, $1 \times 2 \times 3 \times 4 \ldots \times (n-1) \times n = n!$

$n! = n(n-1)!$

$= n(n-1)(n-2)!$ [provided $(n \geq 2)$]

$= n(n-1)(n-2)(n-3)!$ [provided $(n \geq 3)$]

****Permutations** A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

** The number of permutations of n different objects taken r at a time, where $0 < r \leq n$ and the objects do not repeat is $n(n-1)(n-2)\ldots(n-r+1)$, which is denoted by

$$P(n,r) \text{ OR } {}^{n}P_{r} = \frac{n!}{(n-r)!}, 0 \leq r \leq n$$

** ${}^{n}P_{0} = 1 = {}^{n}P_{n}$

** The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r.

** The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is $\frac{n!}{p_1!p_2!\ldots p_k!}$.

** The number of permutations of an dissimilar things taken all at a time along a circle is $(n-1)!$.

** The number of ways of arranging a distinct objects along a circle when clockwise and anticlockwise

arrangements are considered alike is $\frac{1}{2}(n-1)!$.

** The number of ways in which (m + n) different things can be divided into two groups containing m and n things is $\frac{(m+n)!}{m!\,n!}$.

Combination of n different objects taken r at a time, denoted by $^nC_r = \frac{n!}{r!(n-r)!}$.

** $^nP_r = {^nC_r}\, r!$, $0 \le r \le n$

** $^nC_0 = 1 = {^nC_n}$

$^nC_1 = n = {^nC_{n-1}}$

$^nC_2 = \frac{n(n-1)}{2!} = {^nC_2}$

$^nC_3 = \frac{n(n-1)(n-3)}{3!} = {^nC_{n-3}}$

** $^nC_r = {^nC_s} \Rightarrow r = s$ or $r + s = n$

** $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

QUESTIONS FOR HHW

Q.1 If $^{10}Pr=5040$, find the value of r

Q.2 If $5.\, ^4Cr = 6.\, ^5C_{r-1}$, find value of r.

Q.3 How many 6-digit number can be formed from the digits 0,1,3,5,7,9 which are divisible by 10
and no digit is repeated ?

Q.4 How many words can be formed by using the letters of the word ORIENTAL, so that the vowels always occupy the odd places ?

Q.5 How many squares in a chess board?

Q.6 How many palindrome of 5 letters can be made by using letters of the word MATHS?

Q.7 It is required to seat 5 men and 4 women in a row so that the women occupy the even places.
How many such arrangements are possible ?

Q.8 Given12 flags of different colours, how many different signals can be generated if each signal
requires the use of 2 flags, one below the other?

Q.9 There are four bus routes between A and B; and three bus routes between B and C. A man can
travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus

route more than once, in how many ways can he make round trip?
Q.10 In an examination there are three multiple choice questions and each question has 4 choices.
Find the number of ways in which a student can fail to get all answer correct.

Long answer type questions

Q.1 A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl ? (ii) at least one boy and one girl ? (ii) at least 3 girls ?
Q.2 Find the number of words with or without meaning which can be made using all the letters of
the word AGAIN. If these words are written as in a dictionary, what will be the 50th word?
Q.3 In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I
and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions ?
Q.4 How many number of signals that can be sent by 6 flags of different colours taking one or
more at a time?
Q.5 A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and at least
5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

DAY 7(26/10/23)

BINOMIAL THEOREM
MAIN CONCEPTS AND RESULTS

** Binomial theorem for any positive integer n

$$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1}.b + {}^nC_2\, a^{n-2}.b^2 + \dots {}^nC_{n-1}\, a\, b^{n-1} + {}^nC_n\, b^n = \sum_{k=0}^{n} {}^nC_k\, a^{n-k} b^k$$

** The coefficients ${}^nC_r$ occurring in the binomial theorem are known as binomial coefficients.

** There are (n +1) terms in the expansion of $(a+b)^n$, i.e., one more than the index.

** $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_{n-1} x^{n-1} + {}^nC_n x^n$

** $(1-x)^n = {}^nC_0 - {}^nC_1 x + {}^nC_2 x^2 - \dots + (-1)^n\, {}^nC_n x^n$.

** ${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n$.

** ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^{n-1}\, {}^nC_n = 0$.

** General Term in the expansion of $(a+b)^n = t_{r+1} = {}^nC_r a^{n-r}.b^r$

** Middle term in the expansion of $(a+b)^n$

(i) $\left(\frac{n}{2}+1\right)^{th}$ term if n is even (ii) $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+1}{2}+1\right)^{th}$ terms if n is odd.

****Pascal's Triangle**

Index	Coefficients
0	${}^0C_0$ (=1)
1	${}^1C_0$ (=1) ${}^1C_1$ (=1)
2	${}^2C_0$ (=1) ${}^2C_1$ (=2) ${}^2C_2$ (=1)
3	${}^3C_0$ (=1) ${}^3C_1$ (=3) ${}^3C_2$ (=3) ${}^3C_3$ (=1)
4	${}^4C_0$ (=1) ${}^4C_1$ (=4) ${}^4C_2$ (=6) ${}^4C_3$ (=4) ${}^4C_4$ (=1)
5	${}^5C_0$ (=1) ${}^5C_1$ (=5) ${}^5C_2$ (=10) ${}^5C_3$ (=10) ${}^5C_4$ (=5) ${}^5C_5$ (=1)

QUESTIONS FOR HHW

1	$(1+4x+4x^2)^{10}$has 10 TERMS B) 11 TERMS C) 20 TERMS D)21 TERMS
2	The ratio of the coefficients of $x^r$and x^{r-1}in $(1+x)^n$is A) $\frac{n+r}{r}$ B) $\frac{n-r+1}{r}$ C) $\frac{n+r-1}{r}$ D) NONE
3	If the coefficient of $x^2$and x^3in the expansion of $(3+mx)^9$are equal , then the value of m is A) $-\frac{9}{7}$ B) $-\frac{7}{9}$ C) $\frac{9}{7}$ D) $\frac{7}{9}$
4	The term independent of x in the expansion of $\left(2x+\frac{1}{3x^2}\right)^6$is A) 2^{nd} B) 3^{rd} C) 4^{th} D) 5^{th}
5	Using binomial theorem, evaluate $(99)^5$.
6	Expand $\left(x^2+\frac{3}{x}\right)^4$,$x\neq0$b using Pascal triangle.
7	Prove that: $(a+b)^6-(a-b)^6$is aneven number if a and b are integers:
8	Find 4^{th} term of the expansion $\left(3x+\frac{2}{x}\right)^6$
9.	Find 5^{th} term from the end of the expansion $(a+bx)^7$
10.	Find $(a+b)^6-(a-b)^6$hence evaluate$(\sqrt{3}+\sqrt{2})^6-(\sqrt{3}-\sqrt{2})^6$
11	Find a, if the 4th and 5th term of the expansion $(2 + a)^7$ are equal.
12	Expand: $(x2+ 1 - 2x)^3$
13	Find the value of $(a^2+\sqrt{a^2-1})^4+(a^2-\sqrt{a^2-1})^4$
14	Using binomial theorem prove that 5^{4n} +52n-1 is divisible by 676 $\forall$ $n\in N$
15	Find he middle term in the expansion $\left[\frac{x}{7}-\frac{5}{x}\right]^6$
16	Find the coefficient of x^4in $\left[2x^2-\frac{3}{x}\right]^5$
17	Using the binomial theorem, show that 6^n–5n always leaves remainder 1 when divided by 25

18	Find a if coefficients of x^2 and x^3 in $(3+ax)^9$ are equal.
19	The coefficients of 2nd and 3rd terms in the expansion of $(1+a)^n$ are in the ratio 1:2. Find n.
20	Find the middle term(terms) in the expansion of $\left(3x-\frac{x^3}{6}\right)^5$
21	**Case based Question**:In class XI, teacher explained binomial theorem. Two students Shivani and Vishwani trying to solve the exercise. Shivani expanded $(1+x)^6$ by using Binomial theorem, Viswani expanded $(x+1)^6$. **Based on this above information answer the following questions**. 1. According to Shivani find 4th term . 2. Find the value of $6c_0+6c_1+6c_2+6c_3+\ldots\ldots+6c_6$ 3. Find the positive value of x if 3rd terms of Shivani and Vishwani are equal.

DAY 8(27/10/23)

SEQUENCE AND SERIES
CONCEPTS AND RESULTS

** **Sequence :** is an arrangement of numbers in a definite order according to some rule. A sequence can also
be defined as a function whose domain is the set of natural numbers or some subsets of the type {1, 2, 3.... k). ** A sequence containing a finite number of terms is called a finite sequence. A sequence is called infinite if it is not a finite sequence.

** **Series :** If a_1, a_2, a_3,…,an, be a given sequence. Then, the expression $a_1 + a_2 + a_3 +, \ldots + a_n + \ldots$

** **Arithmetic Progression (A.P.) :** is a sequence in which terms increase or decrease regularly by the same
constant.

A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called arithmetic sequence or arithmetic progression if
$a_n + 1 = a_n + d$, $n \in N$, where a_1 is called the first term and the constant term d is called the common
difference of the A.P.

** The n^{th} term (general term) of the A.P. a, a + d, a + 2d, ... is $\mathbf{a_n = a + (n - 1) d}$.

** If a, b, c are in A.P. and $k(\neq 0)$ is any constant, then

(i) a + k, b + k, c + k are also in A.P.

(ii) a – k , b – k , c – k are also in A.P.

(iii) ak, bk, ck are also in A.P

(iv) $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P.

** If a, a + d, a + 2d, …, a + (n – 1) d be an A.P. Then l = a + (n – 1) d.

Sum to n terms $S_n = \frac{n}{2}[2a + (n-1)d$

$= \frac{n}{2}[a + l]$

** **Arithmetic mean (A.M.)** between two numbers a and b is $\frac{a+b}{2}$.

** **n arithmetic means** between two numbers a and b are
$a + \frac{(b-a)}{n+1}$, $a + \frac{2(b-a)}{n+1}$, $a + \frac{3(b-a)}{n+1}$, ... ,$a + \frac{n(b-a)}{n+1}$.

** Sum of n A.M.^{S} = n(single A.M.)

** Three consecutive terms in A.P. are $a - d$, a, $a + d$.
Four consecutive terms in A.P. are $a - 3d$, $a - d$, $a + d$, $a + 3d$.
Five consecutive terms in A.P. are $a - 2d$, $a - d$, a, $a + d$, $a + 2d$.
These results can be used if the sum of the terms is given.

** In an A.P. the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last terms.

** m^{th} term from end of an A.P. = $(n - m + 1)^{th}$ term from the beginning.

****Geometric Progression (G . P.) :A** sequence is said to be a geometric progression or G.P., if the ratio of any term to its preceding term is same throughout.

A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called geometric progression, if each term is non-zero and

$\frac{a_{k+1}}{a_k} = r$(constant) , for $k \geq 1$.

By taking $a_1 = a$, we obtain a geometric progression, a, ar, ar^2, ar^3,…., where a is called the first term
and r is called the common ratio of the G.P.

** **General term of a G .P. =** $a_n = ar^{n-1}$.

** **Sum to n terms of a G .P. =** $\frac{a(r^n - 1)}{r-1}$ if $r > 1$ **and** $\frac{a(1-r^n)}{1-r}$ if $r < 1$.

** **Sum of terms of an infinite G.P. =** $\frac{a}{1-r}$

** **Geometric Mean (G .M.)**: of two positive numbers a and b is the number is $\sqrt{ab}$.

** **n geometric mean** between two numbers a and b are

$\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ $\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$ $\left(\frac{b}{a}\right)^{\frac{3}{n+1}}$ $\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

** Three consecutive terms in G.P. are $\frac{a}{r}$, a, ar .
Four consecutive terms in G.P. are $\frac{a}{r^3}$, $\frac{a}{r}$, ar, ar^3.
Five consecutive terms in G.P. are $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar^2.

These results can be used if the product of the terms is given

QUESTIONS FOR HHW

Q1. If 3^{rd} , 8^{th} and 13^{th} terms of G.P. are p,q and r respectively , then which one of the following is
correct

a . $q^2 = pr$ b. $r^2 = pq$ c. $pq = r$ d. $2q = p+r$

Q2. . If nth term of a sequence is $a_n = (-1)^{n-1} n^3$, then its 9^{th} term is

A) 105   B) 177   C) 324   D) 729

Q3. geometric mean between 1 and 256 IS

A) 8   b. 16   c. 14   d. 12

Q4. If x, 2x+3 ,3x+3 are in G.P. ,then 4^{th} term is

A) -13.5   b. -14.5   c.-15.5   d. -16.5

Q5. Statement I : Four terms of the G.P. $3, 3^2, 3^3, \ldots$ Are needed to give the sum 120

Statement II: $T_n = ar^n$ is n^{th} terms of G.P. whose first term is a and common ratio r.

a) Both the statement I and Statement II are true and statement II is the correct explanation of Statement I
b) Both the statement I and Statement II are true and statement II is not the correct explanation of Statement I
c) Statement I is true but Statement II is false
d) Statement I is false but Statement II is true

Q6 Write first five terms of sequence whose n^{th} term is given by $a_n = (-1)^{n-1} 5^{n+1}$ Q7.for what values of x , the numbers -2/7 ,x -7/2 are in G.P.

Q8. Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and the common ratio is 2.

Q9. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and
the terms.

Q10. Find the sum to n terms of the sequence, 8, 88, 888, 8888… .

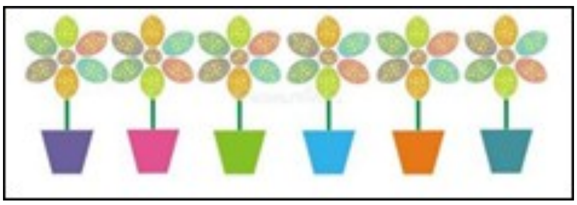
11. Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that
$P^2R^n = S^n$.

12. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

13 **Case study**

Rahul being a plant lover decides toopen a nursery and he bought fewplants with pots. He wants
to place potsin such a way that number of pots infirst row is 2, in second row is 4 and in third row is 8 and so on. Answer the following questions based on the above information.

(i) Find the number of pots in the 8th row.
(ii) Find the total number of pots in 10 rows.
(iii) If Rahul wants to place 510 pots in all, how many rows will be formed?



DAY 9(28/10/23)

STRAIGHT LINES

CONCEPTS AND RESULTS

** Any point on the X-axis is (x, 0) and on the Y-axis is (0, y)

** Distance between two points $A(x_1, y_1)$ & $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

** **Section formula**

(i) Coordinates of a point dividing the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ internally in the ratio m : n is

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

(ii) Coordinates of a point dividing the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ externally in the ratio m : n is

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

** Coordinates of the mid point of the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ is

$$\left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$$

** Centroid of a ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

** In centre of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ is

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$ where a = BC, b = AC, c = AB.

** Area of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3) = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

** Equation of any line parallel to X-axis is y = a, & equation of X-axis is y = 0.

** Equation of any line parallel to Y-axis is x = b & equation of Y axis is x = 0.

** Slope of line inclined at an angle θ with the + ve X- axis = tan θ.

** Slope of a line parallel to X-axis = 0 , slope of a line parallel to Y-axis = undefined.

Slope of a line equally inclined to the coordinate axes is –1 or 1.

** Slope of a line joining the points $A(x_1, y_1)$, $B(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$.

** Slope of the line ax + by + c = 0, is $-\frac{a}{b}$.

** If two lines are parallel, then their slopes are equal.

** If two lines are perpendicular, then the product of their slopes is –1.

** Any equation of the form Ax + By + C = 0, with A and B are not zero, simultaneously, is called the general linear equation or general equation of a line.

(i) If A = 0, the line is parallel to the x-axis (ii) If B = 0, the line is parallel to the y-axis

(iii) If C = 0, the line passes through origin.

** Equation of a line having slope = m and cutting off an intercept '**c**' and Y-axis is y = mx + c.

** Equation of a line through the point (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.

** Equation of a line making intercepts of 'a' & 'b' on the respective axes is $\frac{x}{a} + \frac{y}{b} = 1$

** The equation of the line having normal distance from origin p and angle between normal and the

positive x-axis ω is given by $x \cos\omega + y \sin\omega = p$.

** Distance of a point $P(x_1, y_1)$ from the line ax + by + c = 0 is $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

** Equation of the line parallel to ax + by + c = 0 is ax + by + λ = 0.

** Equation of the line perpendicular to ax + by +c= 0 is bx – ay + λ = 0.

** If two lines are intersecting and θ is the angle between them, then $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

where m_1 = slope of first line, m_2 = slope of second line and θ = acute angle.

If tan θ = negative ⇒ θ = obtuse angle between the intersecting lines.

** Distance between two parallel lines $ax + by + c_1 = 0$ & $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

QUESTIONS FOR HHW

1) Slope of a line which cuts off intercepts of equal lengths on the axes is

(a) -1 (b) 0 (c) 2 (d) $\sqrt{3}$

2.The value of y so that the line through (3,y) and (2,7) is parallel to the line through (-1,4) and (0,6) is

a)7 (b) 10 (c) 9 (d) 8

3) The radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$ is

a) 8 (b) 10 (c) 9 (d) 7

4) The focus of the parabola $y^2 = -8x$ is

A) (2,0) b) (-2,0) c) (0,2) d) (0,-2)

5) Assertion (A). The slope of a line passing through two points (-5, 2) and (3,-2) is $\frac{-1}{2}$

Reason (R). The slope of a line passing through two given points (x_1, y_1) and (x_2, y_2) is $\frac{x_2 - x_1}{y_2 - y_1}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not correct explanation of A.

c) A is true but R is false

d) A is false but R is true.

e) Both A and R are false.